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Abstract

We calculate the inelastic scattering cross sections to one- and two-phonon states in heavy ion collisions. Both Coulomb and nuclear excitations are included and their interplay is studied. Starting from a microscopic approach based on RPA, we go beyond it in order to treat anharmonicities and take explicitely into account the mixing of two-phonon states among themselves and with one-phonon states. Non linear terms in the exciting field are also introduced. These anharmonicities and non linearities are shown to have important effects on the cross sections both in the low energy part of the spectrum and in the energy region of the Double Giant Dipole Resonance.

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1 Introduction

The experimental cross sections for high energy Coulomb excitation in the energy region of the Double Giant Dipole Resonance (DGDR) are systematically larger than the theoretical values obtained by using the standard approach, where the two-phonon spectrum is assumed to be harmonic and the excitation operator linear in the phonon creation and annihilation operators. The discrepancy with experiments ranges from 30% in the case of ^{208}Pb excitation up to a factor of at least 2 for ^{136}Xe [1–6]. Several theoretical studies have been devoted to this problem [7–12]. Recently, we have pointed out a possible explanation of this discrepancy: in the case of large amplitude vibrations like Double GR (DGR), anharmonicities in the internal hamiltonian and non linearities in the external field can become important. In order to see the effects of these new terms on the excitation probabilities of DGR, schematic [10,11] as well as microscopic [12] models have been studied.

In ref. [10] a simple model, namely that of an anharmonic oscillator, was used to study the effects of anharmonicities on the Coulomb excitation of a DGR. In particular, it was applied to study the excitation of ^{136}Xe in the reaction ^{136}Xe + ^{208}Pb at E/A=700 MeV. Adjusting the parameters of the model so that the energy E_2 of the second excited state $|\Psi_2>$ is lower than the double of the energy E_1 of the first excited state $|\Psi_1>$ by about 2 MeV, which is the experimental value, an increase of the cross section associated with $|\Psi_2>$ by 70% was found. The same result was found in ref. [11] within a very similar model. In ref. [10] the excitation probabilities were calculated by solving semiclassical coupled channels equations, i.e. integrating along classical trajectories, which at the very high incident energies considered (E/A around 700 MeV) can be assumed to be straight lines. In order to have a physical insight into the results, an analysis based on second order perturbation theory was also presented in [10]. Such qualitative analysis led to the conclusion that the increase of the cross section has two origins : i) the wavefunction of an excited state is a superposition of states with different numbers of quanta. This effect can be estimated by arguments based on sum rules [13] and can be traced back to the fact that E_2 is somewhat different from $2E_1$; ii) this shift gives also rise to a dynamical effect related to the dependence of the excitation amplitude on the energy of the state populated in the process. Since it depends on the reaction time, the latter effect is very different for nuclear and Coulomb excitations. However, as clearly stated in [14], also in the case of Coulomb excitation at $E/A = 1\ GeV$ it is quite important and cannot be neglected. Therefore, calculations of Coulomb excitation cross sections should take into account this dynamical effect.

A basic physical idea underlying the semiclassical theoretical models used to study grazing nucleus-nucleus collisions is that the excitation of one partner (say A) is due to the mean field of the other (B) acting on it. Since the mean field is a one-body operator, the operator responsible for the excitation of nucleus A has the form

$$W(t) = \sum_{\alpha,\beta} W_{\alpha,\beta}(t) a_\alpha^\dagger a_\beta \tag{1}$$

where $W_{\alpha\beta}(t) = <\alpha|U_B(\vec{R}(t))|\beta>$ and U_B is the mean field of nucleus B. The time dependence comes in through the relative distance R between the two nuclei. The sums in eq. (1) are not restricted a priori and run over occupied (h) and unoccupied (p) single particle states in nucleus A. Of course, when W acts on the ground state and if one neglects the correlations present in it, only the ph terms contribute to the excitation processes, while the hh ones give the potential defining the relative motion trajectory. In general, however, pp and hh terms contribute to the mutual excitation. When W is mapped onto a bosonic space and expressed by phonon creation and annihilation operators $Q_\nu^\dagger$ and Q_ν, one easily realizes that the excitation operator contains in a natural way new quadratic terms, besides the linear ones usually considered. Thus new excitation routes are open since one can directly connect states differing by two phonons and different states with the same number of phonons. These new excitation mechanisms are very important when some selection rule forbids a transition through the linear terms. The inclusion of non linear terms in the excitation operator, together with the anharmonicities, within the schematic model of [10] was found to increase further the cross section to the second excited state, bringing it to the double of the value obtained within the standard approach.

All these ingredients have been taken into account in a realistic calculation of Coulomb excitation in [12]. Mixing of two-phonon states among themselves and with one-phonon states was considered within a boson expansion approach [15] with Pauli corrections. The calculations were done by solving semiclassical coupled channel equations, the channels being superpositions of one- and two-phonon states. Therefore, the positions of the peaks in the cross section are related to the eigenvalues of the internal hamiltonian, including anharmonicities. For the case ^{208}Pb + ^{208}Pb at E/A = 641 MeV an increase of 10% of the cross section in the region around the DGDR was found, bringing the discrepancy with the experimental value down to 18%. It has to be noted that the increase of the cross section in the energy region around the DGDR found in [12] is mainly due to the excitation of several states in that region whose population is strongly suppressed by selection rules when anharmonicities and non-linearities are neglected. Similarly, strong enhancements in the population of two-phonon states at lower excitation energies were also found.

In the above mentioned papers, only Coulomb excitation in relativistic heavy ion collisions was considered. However, the excitation of DGR by the nuclear part of the external field can be important. This is especially true in medium heavy ion collisions at not very high energies (E/A = 20-100 MeV). In fact, the first experimental evidence on the existence of the DGR states were about the double excitation of the ISGQR [2,16], which occurs mainly through the nuclear part of the external field. For heavier systems, in the same bombarding energy region, one may expect that the nuclear and Coulomb parts of the external field play both an important role and interesting effects should arise from their interplay. In the present paper we show the results of calculations including both parts. As we will see, their combined action gives rise to important interference effects on the inelastic cross sections and allows to reach states, mainly of two-phonon character, whose population is strongly suppressed when only one of the two parts is taken into account. Anharmonicities and non linearities are also introduced. It is shown that they strongly affect the cross section to two-phonon states both in the low energy part of the spectrum and in the DGDR energy region. Their importance in the study of heavy ion collisions is therefore confirmed.

2 The approach

In this section we recall the origins of anharmonicities and non linearities and the main points of the approach used for treating them. For details see ref.[12]

The best suited microscopic theory to describe collective excitations in nuclei is RPA, where one introduces operators

$$q_\nu^\dagger = \sum_{p,h}(X^\nu_{ph} a_p^\dagger a_h - Y^\nu_{ph} a_h^\dagger a_p) \tag{2}$$

such that the excited states are

$$|\Psi_\nu >= q_\nu^\dagger |\Psi_0 > \tag{3}$$

and the ground state satisfies to

$$q_\nu |\Psi_0 >= 0 \tag{4}$$

In eq.(2) p (h) denotes single particle occupied (unoccupied) states with respect to the Hartree-Fock ground state $|HF>$. The RPA can be seen as the lowest order of a boson expansion ([17,18])

$$a_p^\dagger a_h \to B_{ph}^\dagger + (1-\sqrt{2})\sum_{p'h'} B^\dagger_{p'h'} B_{p'h} B_{ph'} + \ ... \tag{5}$$

where the operators $B_{ph}^\dagger$ and B_{ph} satisfy boson commutation relations. The terms in (5) after the first one correct for the Pauli principle. When only the lowest order term of eq. (5) is retained, the RPA hamiltonian can be written as

$$H_{RPA} = E_{RPA} + \sum_{\nu} E_{\nu} Q_{\nu}^{\dagger} Q_{\nu} \tag{6}$$

where $Q_{\nu}^{\dagger}$ and Q_{ν} are the bosonic images of the $q_{\nu}^{\dagger}$ and q_{ν} (2) fermionic operators and have the same form as in eq. (2) with $B^{\dagger}$ and B replacing the ph operators. Eq. (6) clearly shows that the excitation spectrum is harmonic in RPA. Already the inclusion of higher order terms in the mapping of eq. (5) leads to an anharmonic hamiltonian. We remark that at the RPA level, only $V_{ph,p'h'}$ and $V_{pp',hh'}$ terms of the residual interaction are taken into account. Including also the other terms $V_{pp',p''p'''}$, $V_{hh',h''h'''}$, $V_{pp',p''h}$ and $V_{ph,h',h''}$ and introducing the mappings [17]

$$\begin{array}{l} a_p^{\dagger} a_{p'} \longrightarrow (a_p^{\dagger} a_{p'})_B = \sum_h B_{ph}^{\dagger} B_{p'h} \\ a_h a_{h'}^{\dagger} \longrightarrow (a_h a_{h'}^{\dagger})_B = \sum_p B_{ph}^{\dagger} B_{ph'} \end{array} \tag{7}$$

one ends up with a hamiltonian containing cubic, quartic, etc, terms in the phonon creation and annihilation operators. We truncate the bosonic image of the hamiltonian to the fourth order, includind up to this order the anharmonicities coming from the corrections for the Pauli principle and from the extra terms in the residual interaction. We restrict ourselves to the space spanned by one- and two-phonon states. Therefore, the eigenstates of the hamiltonian are

$$|\Phi_{\alpha}> = \sum_{\nu} c_{\nu}^{\alpha} |\nu> + \sum_{\nu_1 \nu_2} d_{\nu_1 \nu_2}^{\alpha} |\nu_1 \nu_2> \tag{8}$$

and the corresponding eigenvalues do not form a harmonic spectrum.

As for the excitation operator, in the standard approach only the ph terms of eq. (1) are taken and their lowest order boson expansion is considered. Therefore one gets a linear form in the phonon creation and annihilation operators. When the pp and hh terms are also included, their mappings according to eq.s (7) lead to a quadratic form. These new terms will be called non linear in what follows. Expressing the $B^{\dagger}$ and B operators by the $Q^{\dagger}$ and Q ones, the external field can be written as

$$W = W^{00} + \sum_{\nu} W_{\nu}^{10} Q_{\nu}^{\dagger} + h.c. + \sum_{\nu\nu'} W_{\nu\nu'}^{11} Q_{\nu}^{\dagger} Q_{\nu'} + \sum_{\nu\nu'} W_{\nu\nu'}^{20} Q_{\nu}^{\dagger} Q_{\nu'}^{\dagger} + h.c. \tag{9}$$

The first term in eq. (9) represents the interaction of the two colliding nuclei in

Table 1
One-phonon basis for the nucleus ^{208}Pb. For each state its spin and parity, isospin, energy and percentage of the EWSR are reported.

Phonons	J^π	T	$E(MeV)$	$\%EWSR$
GMR_1	0^+	0	13.610	61
GMR_2	0^+	0	15.022	28
GDR_1	1^-	1	12.435	63
GDR_2	1^-	1	16.662	17
2^+	2^+	0	5.545	15
$ISGQR$	2^+	0	11.599	76
$IVGQR$	2^+	1	21.815	45
3^-	3^-	0	3.464	21
$HEOR$	3^-	0	21.302	37

their ground state. The two next terms connect states of the target differing by one phonon, the fourth term couples excited states with the same number of phonons, while the two last ones allow transitions from the ground state directly to two-phonon configurations. All of them are calculated by double-folding the Coulomb and nuclear nucleon-nucleon interactions with the Hartree-Fock ground state density of the projectile and then with the ground state density or the transition densities of the considered excited states of the target. For the nuclear part we take an M3Y interaction [19]. All matrix elements depend on time through the relative motion trajectory, which we assume to be determined by the Coulomb field alone. The effects of the nuclear interaction on the relative motion are small for grazing collisions and we neglect them.

3 Results and discussion

The above described formalism has been applied to the collision ^{208}Pb on ^{208}Pb at E/A = 50 MeV. The results we are going to present are based on HF+RPA calculations with Skyrme interaction SGII [20]. The transition densities are calculated with the RPA wavefunctions. The excited one-phonon states we have included, their energies and fractions of the Energy Weighted Sum Rule (EWSR) are listed in table 1. We stress that the EWSR's are calculated directly by using the RPA energies and transition probabilities. Only the most collective one-phonon states, exhausting at least 5% of the relevant EWSR, are taken into account. All possible two-phonon states that can be constructed from them, with all possible values of the total angular momentum L, are included. Since these states are always dominated by one component, in the following we denote them by the name of the dominant component (see ref. [12]).

The cross section is calculated, non-perturbatively, by solving the Schrödinger equation in the space of the ground state and the $|\Phi_\alpha>$ states. We then solve the set of linear differential equations for the time dependent amplitudes and construct the cross sections as described in ref. [12]. We have assumed a sharp cutoff transmission coefficient and have integrated the probability of exciting the state $|\Phi_\alpha>$ from a minimum impact parameter whose value has been chosen equal to 13 fm, corresponding to $1.1 \times (A_1^{\frac{1}{3}} + A_2^{\frac{1}{3}})$.

In tables 2-8 we report the inelastic scattering cross sections for some relevant states calculated at different levels of approximation.

Let us first discuss the results obtained at the RPA level, by considering only the linear terms of the external field (first lines in the tables), which correspond to the so-called standard calculation. In this case the states are pure one- or two-phonon states and each two-phonon state has an energy equal to the sum of the energies of the one-phonon states from which it is built, independently of the L value. Moreover, only the couplings of the ground state with the one-phonon states and of the latters with the two-phonon states are present. For each state, we show the results with only the Coulomb part of the external field, with only the nuclear part and with both acting together. By comparing the results reported in the three last columns of the tables one can study the interference between Coulomb and nuclear processes and their relative importance. However, this information is mixed with the effects coming from the channel couplings, which are different in the three cases. This happens because several states are mainly excited by only one part of the external field (Coulomb or nuclear). Therefore, the effective couplings are very different in the three cases. For example, the Coulomb cross section to the first monopole state GMR_1 is very small (0.4×10^{-3} mb) while the nuclear one is quite large (41.77 mb). In a perturbative calculation neglecting the coupling to the other channels one should get a value very close to the latter for the Coulomb+nuclear cross section, which on the contrary we find much larger (59.78 mb) showing the importance of the channel couplings. We have carefully checked this point by repeating some calculations with only a few channels.

An interesting general feature of the two-phonon states is that the cross section to the member of each multiplet with highest L is larger than for the other members. Again, this is due to the fact that, in the case we are now discussing, two-phonon states can only be reached by a two step process.This, together with the existence of selection rules, explains also why we obtain an appreciable cross section to the two-phonon states built with one monopole phonon and the GDR only when the whole interaction is switched on. Indeed, the isovector

Table 2
Inelastic scattering cross sections, in mb, for the system ^{208}Pb + ^{208}Pb at E/A = 50 MeV, for some relevant exited states with angular momentum L=0. Their energies are shown in the second column. The last three columns correspond to the calculation done with only the Coulomb interaction, with only the nuclear one and with both, respectively. For each state three different calculations are reported: the first row corresponds to the harmonic and linear one; the second row refers to the calculation done with only anharmonicities while the external field is linear; the third one is the result of a calculation done with both anharmonicities and non-linearities included.

States ($L=0$)	E (MeV)	Coul.	Nucl.	Coul. + Nucl.	
$3^- \otimes 3^-$	6.93	0.05	17.40	6.78	har. & lin.
	7.89	0.15	23.48	15.38	anhar. & lin.
	"	0.50	25.12	19.71	anhar. & non-lin.
$2^+ \otimes 2^+$	11.09	0.07	2.99	0.51	
	11.23	0.05	2.91	0.50	"
	"	0.02	3.21	0.56	
GMR_1	13.61	0.00	41.77	59.78	
	13.42	0.06	39.78	57.63	"
	"	0.51	41.98	73.52	
GMR_2	15.02	0.00	7.05	11.02	
	14.78	0.01	4.29	7.25	"
	"	0.16	4.02	7.02	
$2^+ \otimes ISGQR$	17.14	0.03	7.36	1.61	
	17.44	0.03	10.42	2.77	"
	"	0.04	12.13	3.14	
$ISGQR \otimes ISGQR$	23.20	0.00	4.15	1.18	
	23.20	0.00	4.51	1.29	"
	"	0.01	5.72	1.29	
$GDR_1 \otimes GDR_1$	24.87	0.03	0.01	0.78	
	24.91	0.02	0.01	0.78	"
	"	0.03	0.02	1.24	

Table 3
Same as Table 2 but for L=1

States ($L=1$)	E (MeV)	Coul.	Nucl.	Coul. + Nucl.	
$2^+ \otimes 3^-$	9.01	0.28	22.05	6.05	har. & lin.
	9.20	0.70	21.30	6.46	anhar. & lin.
	"	4.49	20.37	10.35	anhar. & non-lin.
GDR_1	12.44	241.47	7.51	297.36	
	12.31	246.98	7.18	324.08	"
	"	254.24	6.43	331.86	
GDR_2	16.66	7.68	0.63	5.13	
	16.61	7.24	0.61	5.30	"
	"	6.84	0.69	3.36	
$ISGQR \otimes 3^-$	15.06	0.08	27.73	8.42	
	15.14	0.33	29.22	9.73	"
	"	0.53	26.74	8.96	
$GDR_1 \otimes 2^+$	17.98	0.08	0.20	1.76	
	17.77	0.34	0.24	1.59	"
	"	0.36	0.34	2.03	
$GDR_1 \otimes ISGQR$	24.03	0.02	0.23	1.76	
	24.07	0.06	0.25	1.94	"
	"	0.05	0.32	2.89	
$GMR_1 \otimes GDR_1$	26.05	0.00	0.50	4.21	
	26.06	0.01	0.44	3.47	"
	"	0.01	0.44	4.21	
$GMR_2 \otimes GDR_1$	27.46	0.00	0.11	0.91	
	27.45	0.01	0.11	0.93	"
	"	0.01	0.13	1.35	

character of the GDR forbids its excitation by the nuclear field, while the Coulomb excitation of monopole states is strongly suppressed. Therefore, only a two step process, one involving the nuclear excitation of the GMR and the other one the Coulomb excitation of the GDR, can populate such states.

Let us now concentrate on the DGDR region. By summing the cross sections to the two-phonon states in the energy range between 22 MeV and and 30

Table 4
Same as Table 2 but for L=2

States ($L=2$)	E (MeV)	Coul.	Nucl.	Coul. + Nucl.	
2^+	5.55	558.42	80.82	422.01	har. & lin.
	5.18	644.20	73.94	483.54	anhar. & lin.
	"	598.20	66.70	451.15	anhar. & non-lin.
$3^-\otimes 3^-$	6.93	0.29	25.86	10.3	
	7.31	2.04	19.46	8.45	"
	"	1.25	14.63	7.21	
$2^+\otimes 2^+$	11.09	0.37	2.90	0.60	
	11.27	2.55	7.10	3.64	"
	"	2.62	7.12	3.97	
ISGQR	11.60	255.77	122.55	176.48	
	11.59	287.43	125.63	195.04	"
	"	367.12	119.83	240.01	
$GDR_1\otimes 3^-$	15.90	0.46	0.61	5.64	
	15.94	0.26	0.69	4.93	"
	"	0.19	0.62	4.30	
$2^+\otimes ISGQR$	17.14	0.22	7.33	1.43	
	17.31	0.20	9.05	2.40	"
	"	0.17	9.51	2.77	
$GMR_1\otimes 2^+$	19.16	0.00	10.28	3.88	
	19.15	0.00	10.03	3.90	"
	"	0.01	15.21	6.86	
$IVGQR$	21.81	2.75	1.91	8.87	
	21.69	2.80	1.64	9.55	"
	"	2.98	1.51	8.17	
$ISGQR\otimes ISGQR$	23.20	0.04	4.29	1.08	
	23.23	0.10	5.05	1.51	"
	"	0.12	5.64	1.42	
$GDR_1\otimes GDR_1$	24.87	1.70	0.03	3.76	
	24.68	1.65	0.11	3.28	"
	"	1.63	0.11	4.65	

Table 5
Same as Table 2 but for L=3

States ($L=3$)	E (MeV)	Coul.	Nucl.	Coul. + Nucl.	
3^-	3.46	149.74	228.56	253.35	har. & lin.
	3.21	140.39	230.06	252.67	anhar. & lin.
	"	112.16	233.50	236.65	anhar. & non-lin.
$2^+ \otimes 3^-$	9.01	0.46	13.91	3.39	
	9.17	0.56	16.67	5.68	"
	"	0.59	17.84	5.61	
$ISGQR \otimes 3^-$	15.06	0.24	17.86	5.04	
	15.17	0.23	17.34	5.55	"
	"	0.16	18.50	4.45	
$GMR_1 \otimes 3^-$	17.07	0.00	31.14	15.26	
	17.25	0.09	29.44	14.17	"
	"	0.06	28.70	13.08	
$GDR_1 \otimes 2^+$	17.98	10.07	0.94	6.68	
	17.98	8.66	0.88	6.13	"
	"	7.74	1.13	8.25	
$GMR_2 \otimes 3^-$	18.49	0.00	7.68	4.09	
	18.58	0.02	9.21	4.71	"
	"	0.01	9.00	3.99	
$HEOR$	21.30	2.09	25.71	24.53	
	21.19	2.37	28.98	26.22	"
	"	2.64	27.12	28.66	
$GDR_1 \otimes ISGQR$	24.03	5.59	1.31	5.86	
	24.03	5.17	1.27	5.95	"
	"	8.46	1.64	9.04	
$GMR_1 \otimes HEOR$	34.91	0.00	2.33	1.39	
	34.96	0.00	1.76	1.16	"
	"	0.00	1.55	1.47	

Table 6
Same as Table 2 but for L=4

States ($L=4$)	E (MeV)	Coul.	Nucl.	Coul. + Nucl.	
$3^-\otimes 3^-$	6.93	0.15	20.21	7.99	har. & lin.
	7.16	0.12	19.88	8.01	anhar. & lin.
	"	0.10	21.64	7.12	anhar. & non-lin.
$2^+\otimes 2^+$	11.09	11.63	9.03	5.52	
	11.25	10.19	8.82	4.89	"
	"	8.98	10.12	4.88	
$GDR_1\otimes 3^-$	15.90	3.22	2.77	16.42	
	15.86	3.37	2.45	16.22	"
	"	2.85	2.62	18.81	
$2^+\otimes ISGQR$	17.14	13.04	25.42	7.29	
	17.07	12.69	28.02	7.35	"
	"	17.92	31.22	9.01	
$ISGQR\otimes ISGQR$	23.20	3.89	18.08	4.85	
	23.26	3.84	19.16	4.74	"
	"	6.19	21.49	5.16	
$GDR_1\otimes HEOR$	33.74	0.07	2.77	1.74	
	33.65	0.06	2.45	1.59	"
	"	0.07	2.62	1.82	

MeV one gets 12.02 mb (Coulomb), 73.25 mb (nuclear) and 47.24 mb (total) respectively in the three cases corresponding to the three columns of the first lines in the tables. Thus we see that the nuclear excitation is dominant, as expected at the bombarding energy considered, and the overall effect of the Coulomb- nuclear interference is destructive. However, it is worthwhile mentioning that this effect is not uniform for all states. On the contrary, for the $|ISGQR\otimes ISGQR>$ states it leads to strong cancellations, while the combined action of the two parts of the external field is constructive in all cases involving the GDR. This effect can be traced back to the peculiar behaviour in the tail of the transition densities for nuclei with a neutron excess [21].

As shown in [10,12] anharmonicities and non linearities strongly modify the Coulomb excitation cross section at high bombarding energy. In order to study their role at lower energies, when both Coulomb and nuclear excitations are important, we have performed two more series of calculations whose results

Table 7
Same as Table 2 but for L=5

States ($L = 5$)	E (MeV)	Coul.	Nucl.	Coul. + Nucl.	
$2^+ \otimes 3^-$	9.01	5.98	43.51	11.27	har. & lin.
	9.06	5.77	42.89	11.01	anhar. & lin.
	"	4.21	45.95	11.12	anhar. & non-lin.
$ISGQR \otimes 3^-$	15.06	3.36	60.20	19.97	
	15.21	3.32	60.28	19.09	"
	"	2.61	66.53	17.26	
$IVGQR \otimes 3^-$	25.28	0.05	0.62	1.26	
	25.23	0.05	0.65	1.41	"
	"	0.05	0.70	0.74	
$2^+ \otimes HEOR$	26.85	0.13	4.30	0.77	
	26.81	0.12	4.72	0.84	"
	"	0.17	4.00	0.77	
$ISGQR \otimes HEOR$	32.90	0.09	6.18	1.85	
	32.94	0.09	6.52	1.93	"
	"	0.11	6.08	1.69	

Table 8
Same as Table 2 but for L=6

States ($L = 6$)	E (MeV)	Coul.	Nucl.	Coul. + Nucl.	
$3^- \otimes 3^-$	6.93	0.91	58.02	23.29	har. & lin.
	7.43	0.89	55.83	21.85	anhar. & lin.
	"	0.63	60.31	21.50	anhar. & non-lin.
$3^- \otimes HEOR$	24.77	0.03	11.38	4.81	
	24.78	0.03	11.73	4.87	"
	"	0.05	10.67	5.50	

are reported in the second and third lines of the same tables 2-8.

To disentangle the effects of the anharmonicities alone, let us first consider the case when the the external field is still assumed linear in the phonon creation and annihilation operators while the internal hamiltonian contains the extra terms of the residual interaction listed before eq. (7) together with the corrections for the Pauli principle. Then its eigenstates are superpositions of one-and two-phonon states (see eq. (8)), the eigenvalues are shifted with respect to the harmonic RPA limit and the members of each multiplet are splitted according to the different possible L values. Some details on the results obtained by diagonalizing the anharmonic hamiltonian can be found in ref. [12]. The cross sections for this case are shown in the second lines of the tables where now the states are denoted by their main component. Even with the same linear external field as before, new excitation routes, involving different components of the wavefunctions, are open. As said before, also the shifts and splittings introduced by the anharmonicities affect the cross sections. From the reported values of the energies one can already see that in general, for the ^{208}Pb nucleus we are considering, the shifts and splittings are quite small, namely of the order of a few hundreds KeV. One exception is the case of the states whose main component is the two-phonon configuration built with two low-lying 3^- states. The member of the multiplet most affected by the anharmonic terms is the L=0 one, whose energy is shifted up by almost 1 MeV, from 6.93 MeV to 7.89 MeV. This shift should reduce the cross section with respect to the previously discussed harmonic limit. On the contrary, we obtain an increase from 6.78 mb to 15.38 mb. We can then conclude that the mixing in the wavefunction, whose second main component is the GMR_1 with amplitude -0.163, produces a strong enhancement. On the other hand, the L=6 member is a pure two-phonon state. In this case the 500 KeV shift up in energy leads to a 10% decrease of the cross section. Another interesting case is the L=1 state labelled $|2^+ \otimes 3^- >$, whose Coulomb excitation cross section becomes more than the double when the anharmonicities are taken into account. This is due to the presence of a GDR component in its wavefunction [12]. The cross sections to the states mainly of one-phonon character are also affected by the presence of anharmonicities. In particular, for the monopole states we get a decrease, especially large for the second one, while for the GDR and the ISGQR we get an increase by about 10%. In the DGDR region we find essentially no modifications.

On the contrary, the introduction of the non linear terms in the external field enhances the cross section in that region by 20% (see third lines in the tables), namely from 47.24 mb to 58.86 mb. A closer inspection shows that this enhancement is mainly due to the terms W^{11} which couple states with the same number of phonons (see eq. (9). In order to understand how W^{11} acts, let us consider as an example the population of the $|GDR \otimes GQR >$ state. When W^{11} is absent, the only ways leading to it can be visualized as in fig.1 while the direct transition from the ground state is strongly disfavoured because of the big energy difference.

Several other processes leading to the same final state are made possible by the action of W^{11}, like the ones shown in fig. 2 and others, not necessarily involving only the GDR and the ISGQR. Of course, in general all these processes do not interfere constructively. In the particular example we have just illustrated, the

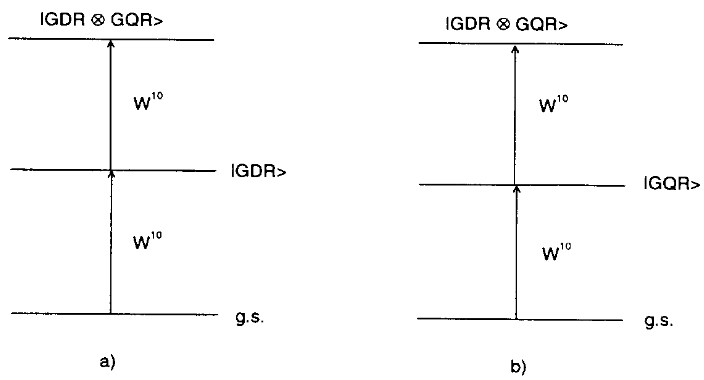

Fig. 1. Schematic representation of the excitation of the $|GDR \otimes GQR >$.

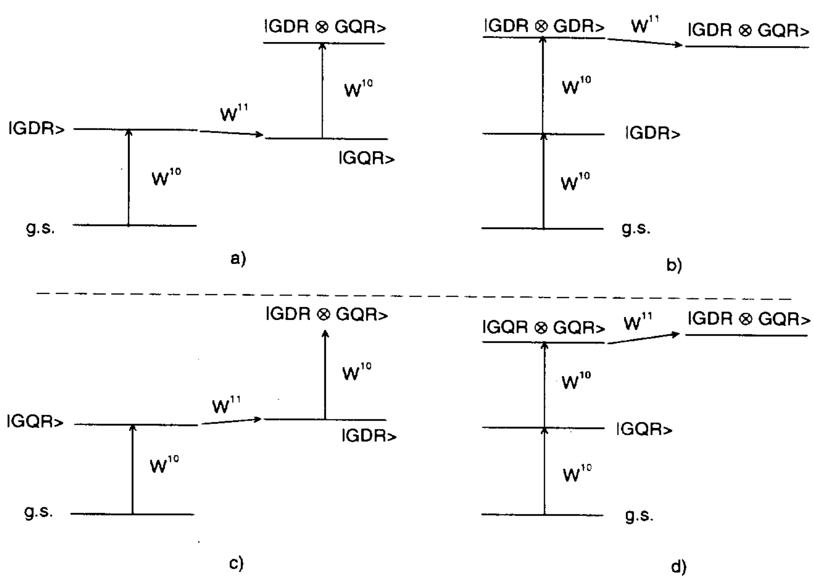

Fig. 2. Schematic representation of the excitation of the $|GDR \otimes GQR >$.

total effect of the inclusion of W^{11} is to increase the cross section by 50% , namely from 7.89 mb to 11.99 mb. Among the other states, the $|2^+ \otimes 3^-; L = 1 >$ one is interesting because it is an example of a direct transition from the ground state to a two-phonon state through the term W^{20} of the external field. In particular, in this case, it is the Coulomb excitation cross section to become much larger. This effect sums up with the previously discussed one, due to the mixing in the wavefunction. A similar enhancement was found in [12] at relativistic bombarding energy.

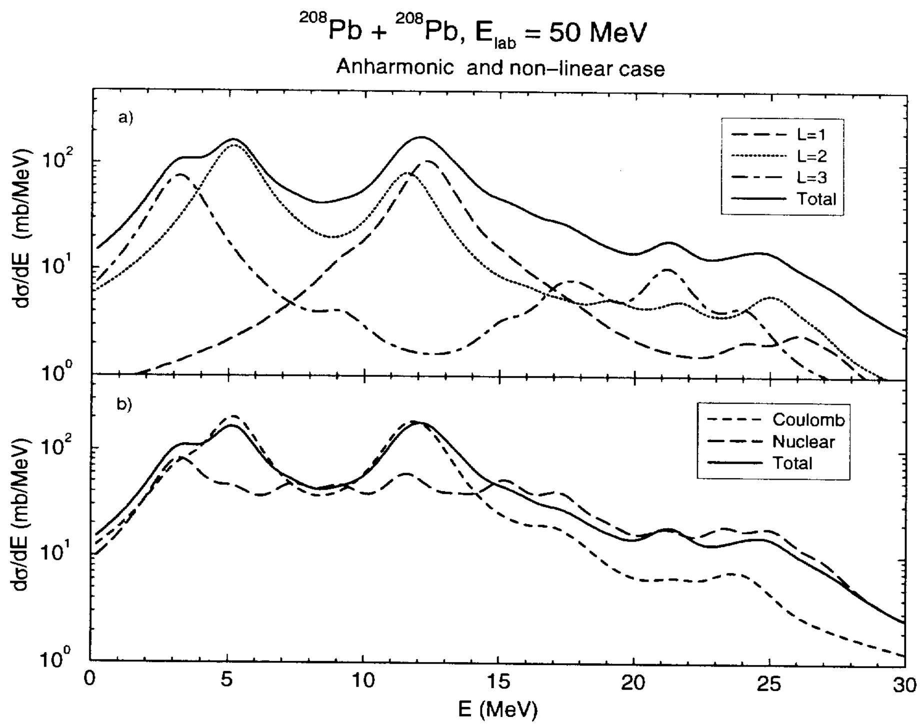

Fig. 3. Inelastic cross section for the system ^{208}Pb+ ^{208}Pb at E_{lab}=50 MeV/A as function of the excitation energy. In the upper part it is shown the contribution of various multipolarities as indicated in the legend. The total cross section corresponds to the sum of all the considered multipolarities included the ones not shown in the figure. In part b) it is plotted the contribution of the nuclear and Coulomb part together with the total.

In order to have a global view of the effects discussed above we have computed the complete inelastic cross section by summing all tge contributions of the various states after a smoothing by a Lorentzian with a width Γ = 2MeV. The result is shown in fig. 3. In the upper part we plot separately the contribution for each angular momentum. We have not shown the L= 4, 5 and 6 $\hbar$ because they give a smaller contribution and also to have an easy readable figure, although the line corresponding to the total cross section includes them. We see that the main contribution in the region of DGDR is given by the quadrupole and octupole multipolarities. In fig. 3.b we plot the nuclear and Coulomb contribution together with their sum. We note that in the energy region below the giant dipole and quadrupole resonances the important contribution comes from the Coulomb one while at higher energies the nuclear part is dominant.

4 Conclusions

We have calculated the inelastic scattering cross sections to one- and two-phonon states for the ^{208}Pb + ^{208}Pb collision at E/A=50 MeV. On one hand we have studied the interplay between Coulomb and nuclear excitation and have shown that the interference effects are very important for such heavy nuclei and moderate bombarding energy. These interferences may lead to rather complicated modifications of the predicted cross section due to their complex influence on the coupling of many channels. On the other hand, we have analyzed the role played by anharmonicities in the excitation spectrum and non linearities in the operator describing the mutual interaction of the collision partners. Focusing our attention on the states whose main component is a two-phonon one, we can draw the following conclusions.

Taking into account both Coulomb and nuclear excitations it is possible to reach states which cannot be populated when only one of these two processes is considered. This is especially evident when anharmonicities and non linearities are neglected, so that two-phonon states can only be populated through a two-step process, and one of the two steps is suppressed by a selection rule. The most striking case is that of the $|GMR \otimes GDR>$ states: the isovector character of the GDR forbids its nuclear excitation while the Coulomb excitation of the GMR is strongly disfavoured.

For the low-lying two-phonon states, we have found that anharmonicities strongly modify the cross section because they can be directly excited from the ground state by W^{10} through their relatively large one-phonon component. The non linear terms in the external field, namely W^{20}, lead also to an appreciable increase of the cross section.

In the DGDR region, i.e. the region between 22 MeV and 30 MeV excitation energy, anharmonicities are quite small in the considered case of ^{208}Pb and the direct transition from the ground state is disfavoured since it involves a big energy jump. However, for some states in this energy region, the term W^{11}, connecting different states with the same number of phonons, produces an enhancement of the cross section. This effect is particularly important for the $|GDR \otimes GQR>$. In this energy region the nuclear contribution to the total inelastic cross section is predominant.

Several other two-phonon states in the DGDR region acquire an appreciable cross section when anharmonicities and non linearities are taken into account. This, together with the effects discussed in the previous point, bring to a 25% enhancement of the cross section.

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